Using viability theory for managing environmental systems

Jean-Denis Mathias *et al*.

INRAE, Laboratoire d'Ingénierie pour les Systèmes Complexes,
9 avenue des landais, 63170 Aubière, France.

Context

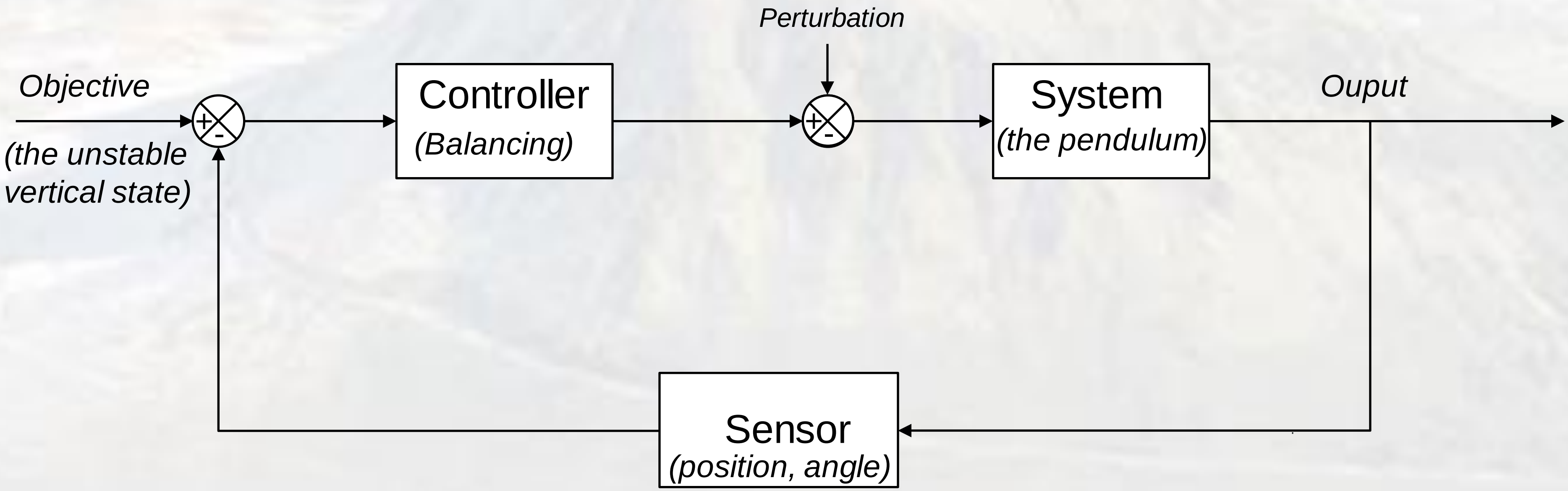
From control engineering to environmental management: some insights

Example: inverted pendulum

Stepan Ozana, 2018, https://www.youtube.com/watch?v=meMWfva-Jio&t=18s

Context

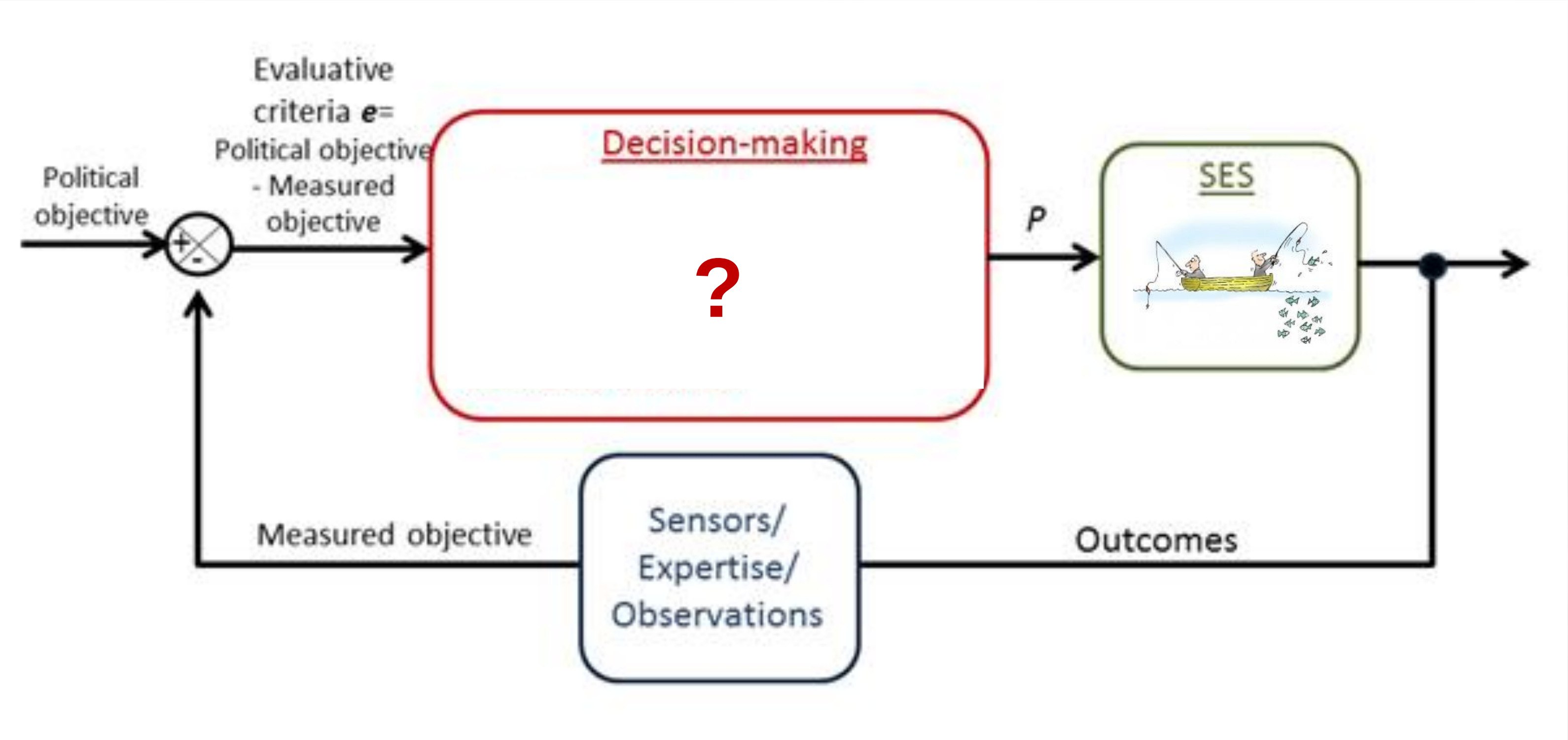
From control engineering to environmental management: some insights



Context

From control engineering to environmental management: some insights

Control engineering in the case of social-ecological systems



Context

Example 1: managing social-ecological systems

Knowledge and decision-making

Harvesting intensity?

Unexpected event?

Carrying capacity?

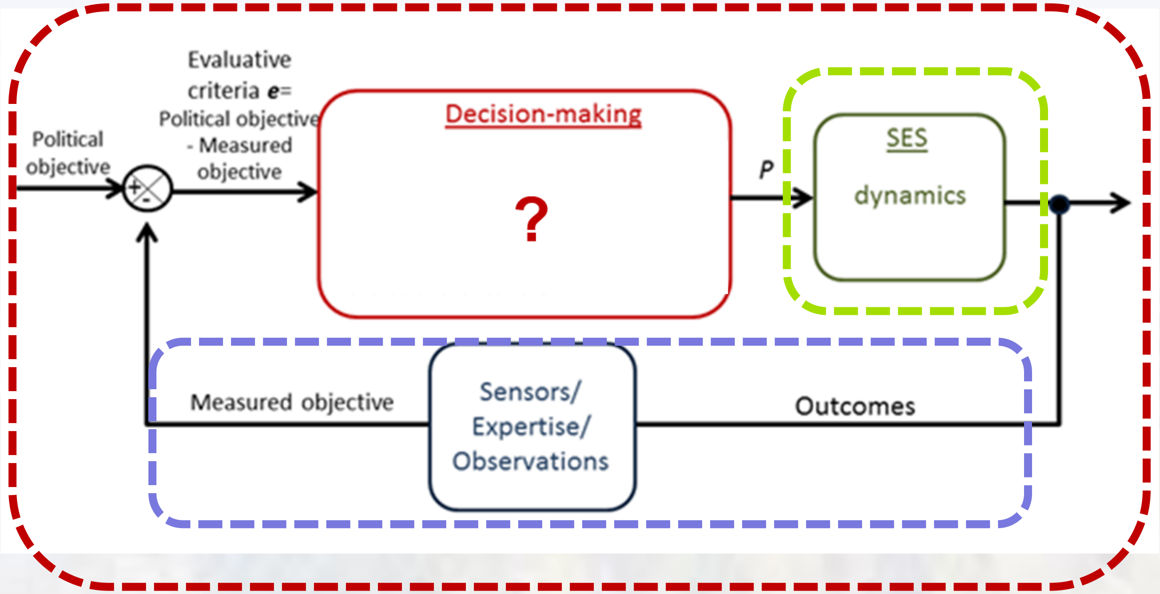
What is the biomass?

Decision-making based on observations

Context

Example 1: managing social-ecological systems

Common strategies in the literature



Omniscience

Viability, optimization

Biological processus

Time series

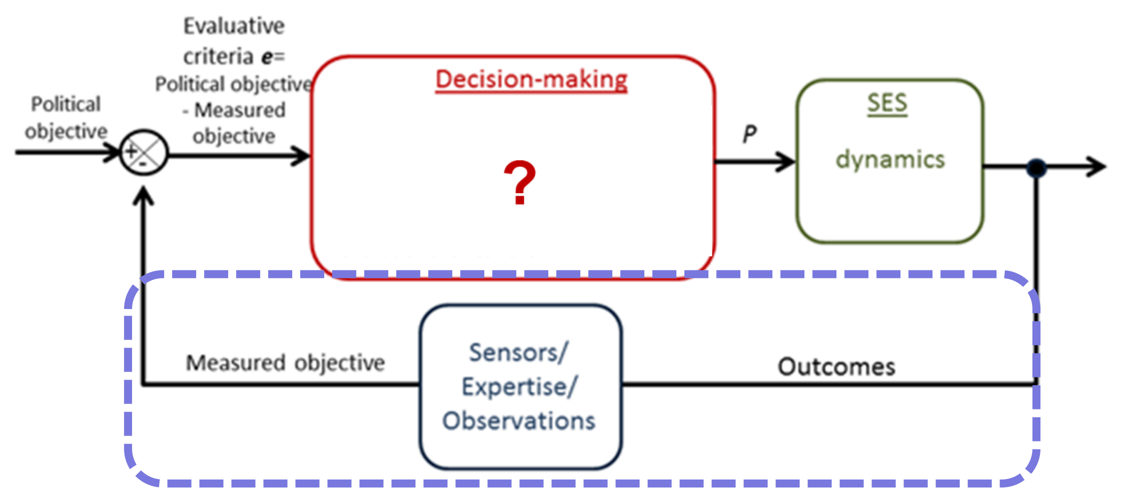
Early warnings

MSY

Context

Example 1: managing social-ecological systems

Observability and controllability of social-ecological systems



Time series

Dynamics

$$\frac{dx}{dt} = F(x) - Y(u,x)$$

Observation

$$z(t)=G(x(t))$$

Question of state observability and controllability

Context

Example 2: the safe operating space as a control problem

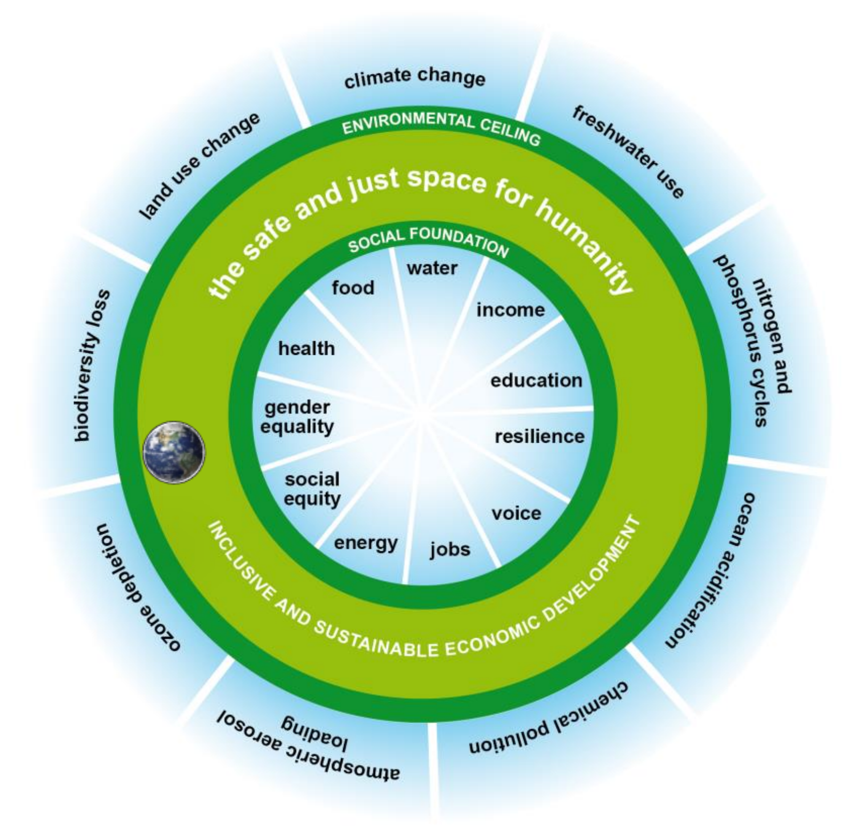
Maintaining Earth system in a « desirable » state…

safe operating space

- Defining desirability from properties of interest
- Defining constraints or limits

Controlled dynamic problem

- Defining the Earth dynamics
- Defining our « control » as our capacity of action

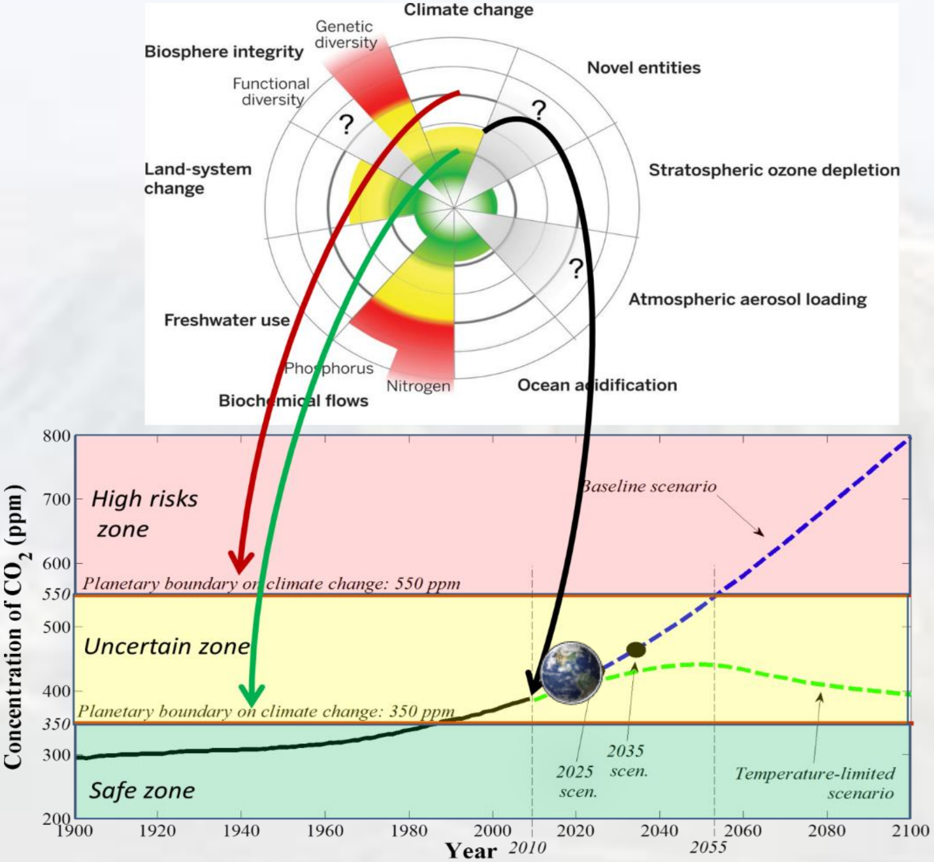


https://chrisriedy.me/2014/01/28/living-in-the-doughnut-of-sustainability/

Context

Example 2: the safe operating space as a control problem

Recovering a desirable state of Earth system ➡ *concept of résilience*



(Rockström et al 2009; Steffen et al 2015)

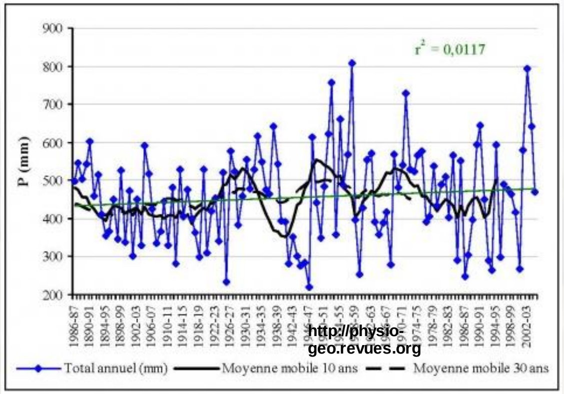
Context

Example 3: clarifying the concept of resilience (with uncertainty)…

Flood in Texas and Oklahoma

Elaine Thompson / The Associated Press

Uncertainty and variations



http://physio-geo.revues.org

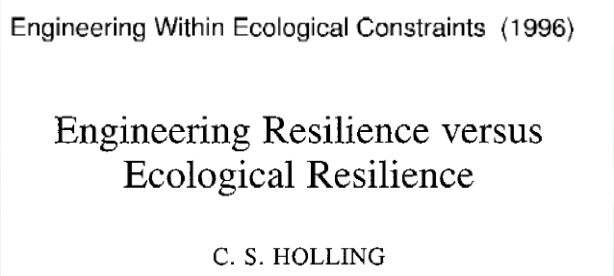
Some difficulties to characterize uncertain dynamics…

How to manage daily variations and extreme events?

Context

Example 3: clarifying the concept of resilience (with uncertainty)…

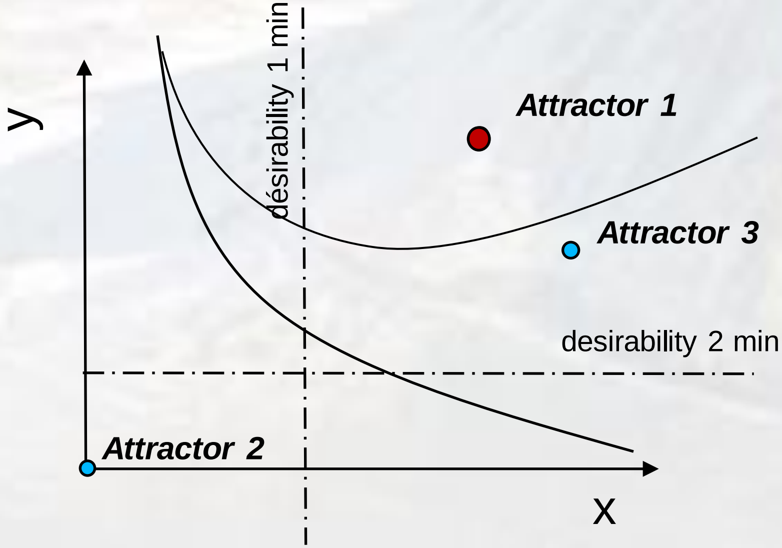
Management of SES, **FOR** the « resilience » of SES…



Engineering Within Ecological Constraints (1996)

Engineering Resilience versus
Ecological Resilience

C. S. HOLLING



The Role of Adaptive Management as an Operational Approach for Resource Management Agencies

Barry L. Johnson

"'…that allows managers to react when conditions change. The result is that, rather than managing for a single, optimal state, we manage within a range of acceptable outcomes while avoiding catastrophes and irreversible negative effects."

Complying with given social-ecological constraints within a dynamical controlled framework.

Outline

I. Concept of resilience

II. Viability theory

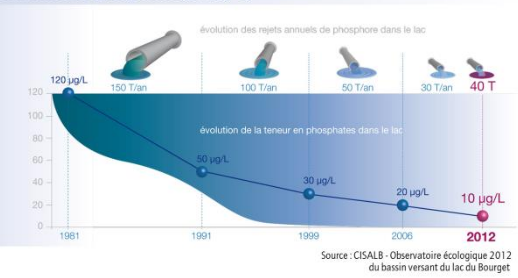
III. Case of lake eutrophication

IV. Conclusions

I Concept of resilience

Case of lake eutrophication

Lake Bourget (France)



Source : CISALB - Observatoire écologique 2012 du bassin versant du lac du Bourget

Impact the quality of the water in terms of clarity, presence of cyanobacteria…

Impact on the environment and on the local economy (tourism, agriculture…)

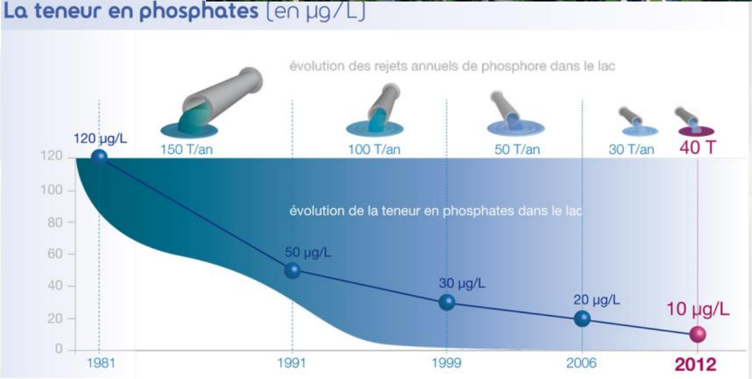
There is a need of (effective) tools for helping policy-makers in their decision process.

I Concept of resilience

Issue 1: Properties of interest

Example of lake eutrophication

Lake Bourget



La teneur en phosphates (en µg/L)

Source : CISALB - Observatoire écologique 2012 du bassin versant du lac du Bourget

Example of a property of interest:

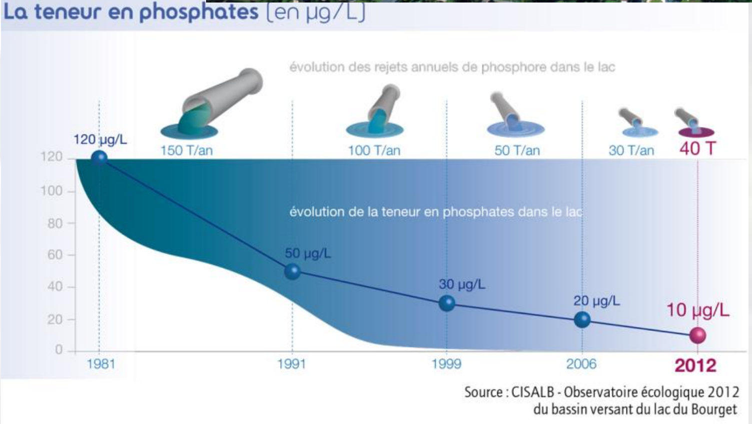
« Having a phosphorus concentration lower than 10 µg/L»

- No optimization, complying with a constraint
- Difficulties to clearly formalize the properties of interest with stakeholders

Issue 2: Dynamical framework

Example of lake eutrophication

Lake Bourget


La teneur en phosphates (en µg/L)

Source : CISALB - Observatoire écologique 2012
du bassin versant du lac du Bourget

Example of a propertie of interest:

« *Having a phosphorus concentration lower than 10 µg/L*»

Evolution in time

Need to describe the evolution of the property of interest (here P(t))

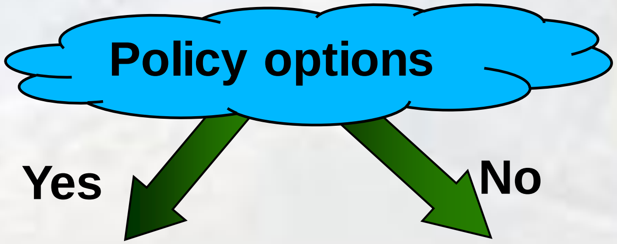
I Concept of resilience

Resilience 1.0

Resilience is defined as the capacity of recovering a property of interest at time horizon T

« Having a phosphorus concentration < 10 µg/L»

Will the phosphorous concentration be lower than 10 µg/L in 5 years ?



Policy options

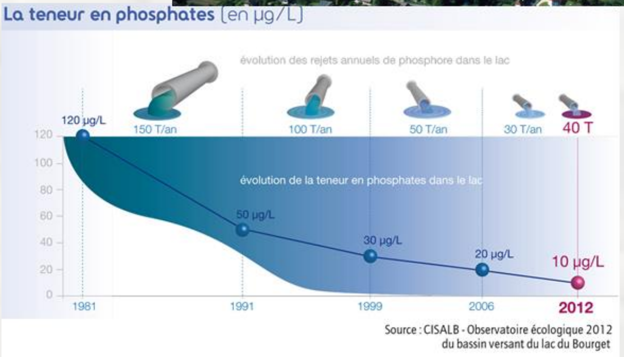
Yes

No

Resilient

Not resilient

Lake Bourget



La teneur en phosphates (en µg/L)

Source : CISALB - Observatoire écologique 2012 du bassin versant du lac du Bourget

No guaranty of keeping the property after recovery…

I Concept of resilience

Resilience 2.0

Resilience is defined as the capacity of recovering a property of interest at time horizon T
AND
keeping it after recovery

« *Having a phosphorus concentration < 10 µg/L*»

Will the phosphorous concentration be lower than 10 µg/L in 5 years **and stay < 10 µg/L** ?

Policy options

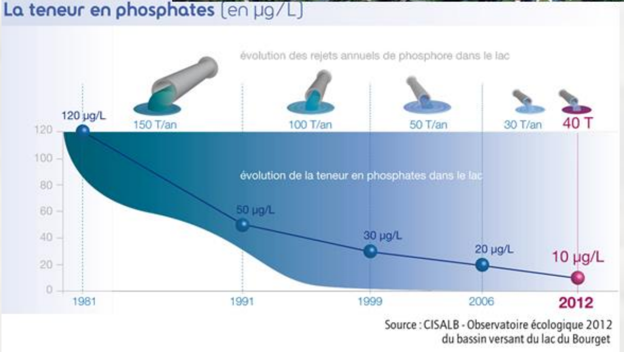
Yes

Resilient

No

Not resilient

Lake Bourget



Source : CISALB - Observatoire écologique 2012 du bassin versant du lac du Bourget

Using the viability theory in order to know if the property is kept (or not) after recovery…

Outline

I. Concept of resilience

II. **Viability theory**

III. Case of lake eutrophication

IV. Conclusions

II. Viability theory

Using viability theory for operationalizing resilience

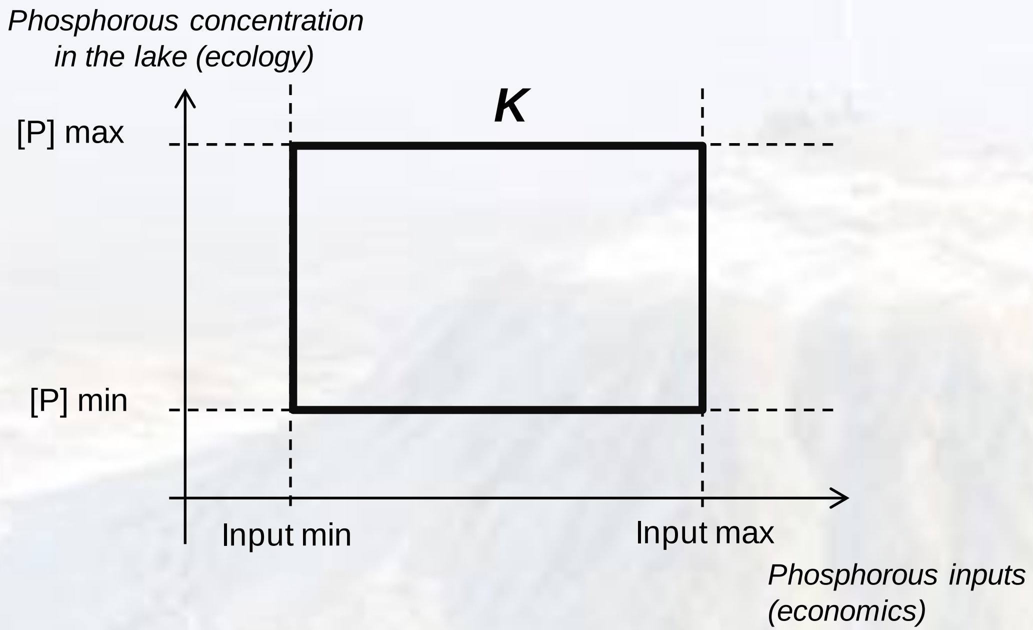
Resilience is defined as the capacity of recovering a property of interest at time horizon T

AND

keeping it after recovery

1. Finding the set of points enabling to keep the properties of interest during a given time horizon. This set will be called the *viability kernel* (Aubin, 1991)
2. Finding the resilient pathways that enable to recover the properties of interest **in the viability kernel (resilience 2.0)**

Viability kernel

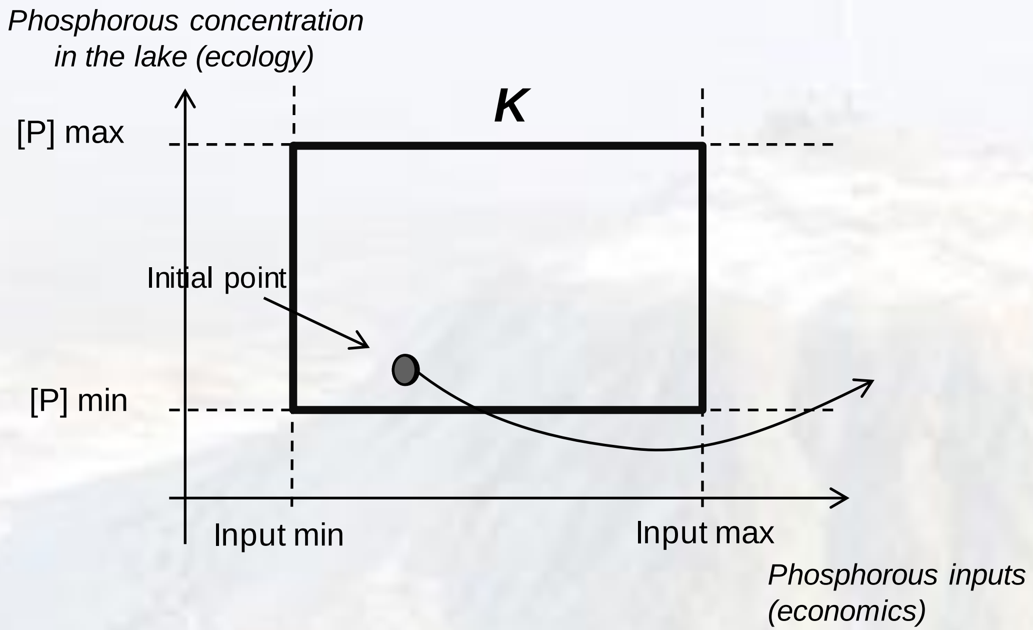


4 properties of interest:

« Having a minimum and a maximum values of phosphorous concentration » (ecological issues)

« Having a minimum and a maximum values of phosphorous inputs » (economic issues)

Viability kernel

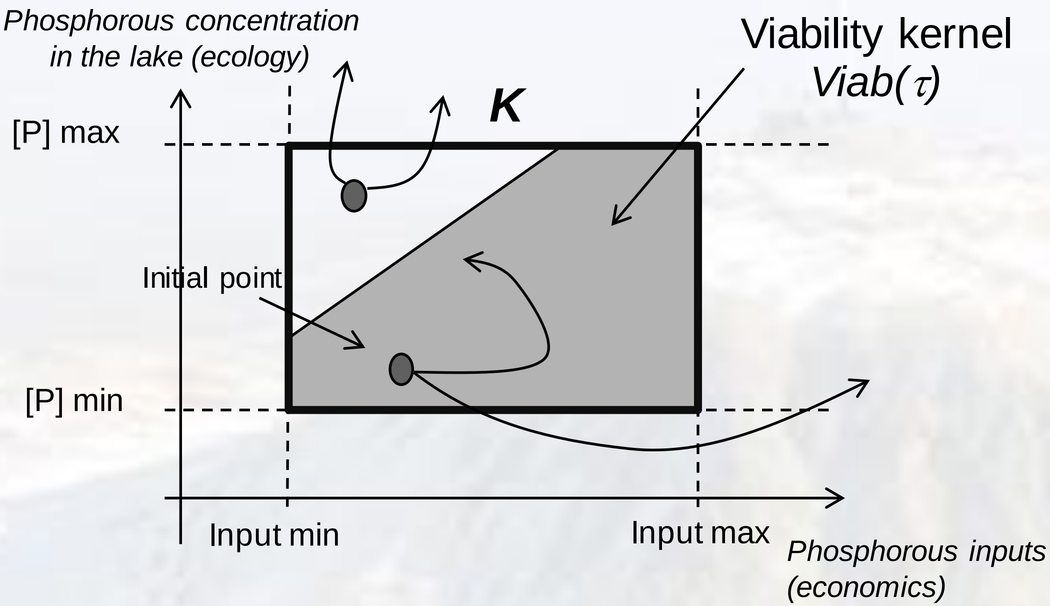


4 properties of interest:

« Having a minimum and a maximum values of phosphorous concentration » (ecological issues)

« Having a minimum and a maximum values of phosphorous inputs » (economic issues)

Viability kernel



4 properties of interest:

« Having a minimum and a maximum values of phosphorous concentration » (ecological issues)

« Having a minimum and a maximum values of phosphorous inputs » (economic issues)

Viab(τ) is the set of initial points (in ***K***) such as there is, at least, a policy that enables to stay in ***K*** during the time horizon τ

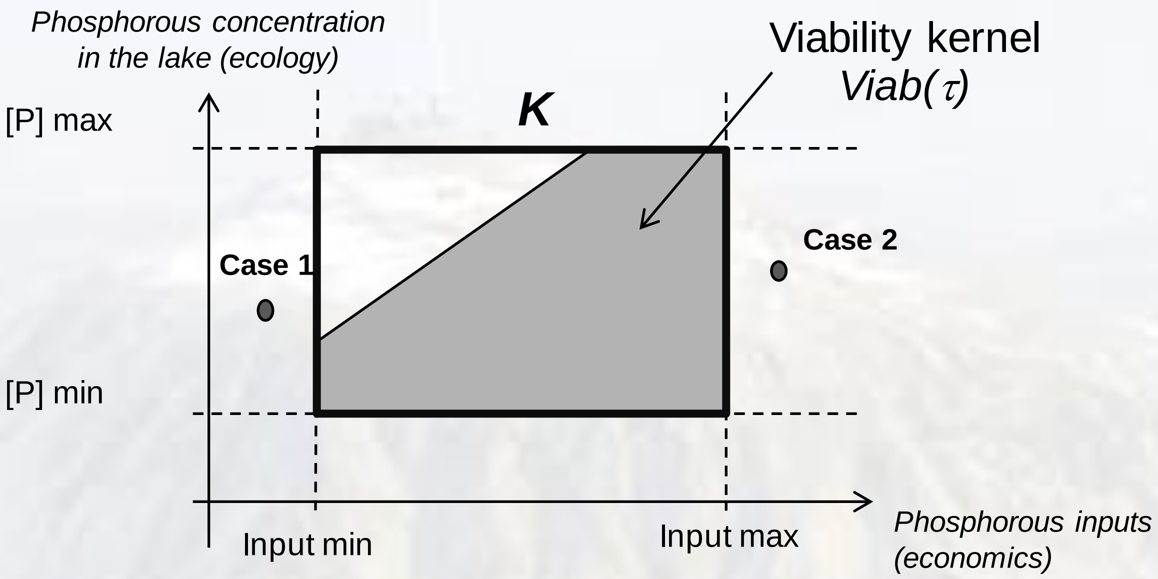
$$\boxed{\mathrm{Viab}(\tau) = \{\, x_0 \in K \mid \exists f \in F(\tau),\ \forall t \leq \tau,\ x_t = g_f(t, x_0) \in K \}}$$

II. Viability theory

Resilience 2.0

Case of an (extreme) event

- Case 1: economic crisis;
- Case 2: pollution



Phosphorous concentration in the lake (ecology)

Viability kernel *Viab(τ)*

[P] max

K

Case 1

Case 2

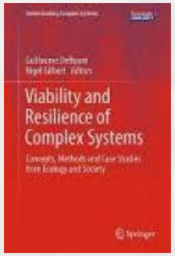
[P] min

Input min

Input max

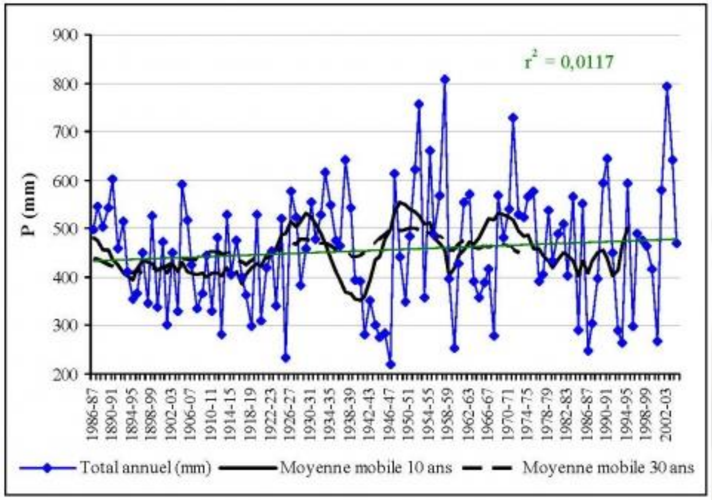
Phosphorous inputs (economics)

Case 1: resilience 1.0 but not resilience 2.0;
Case 2: resilience 2.0 (guarantee of keeping the properties): this is the basin of attraction of *Viab(τ)* (PhD Sophie Martin, european project PATRES (2007-2010))



Viability and Resilience of Complex Systems

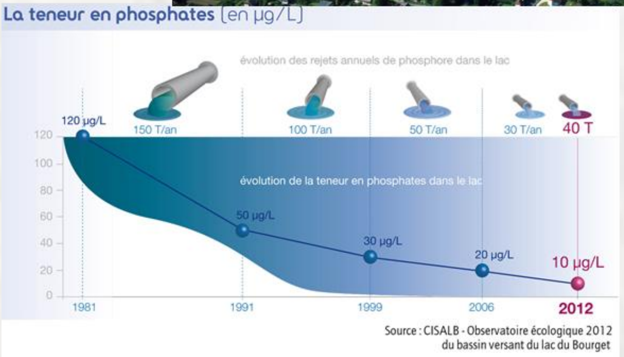
Case of variations in phosphorous inputs



Lack of knowledge

Modeled by a stochastic process
or probabilistic variable

Lac du Bourget

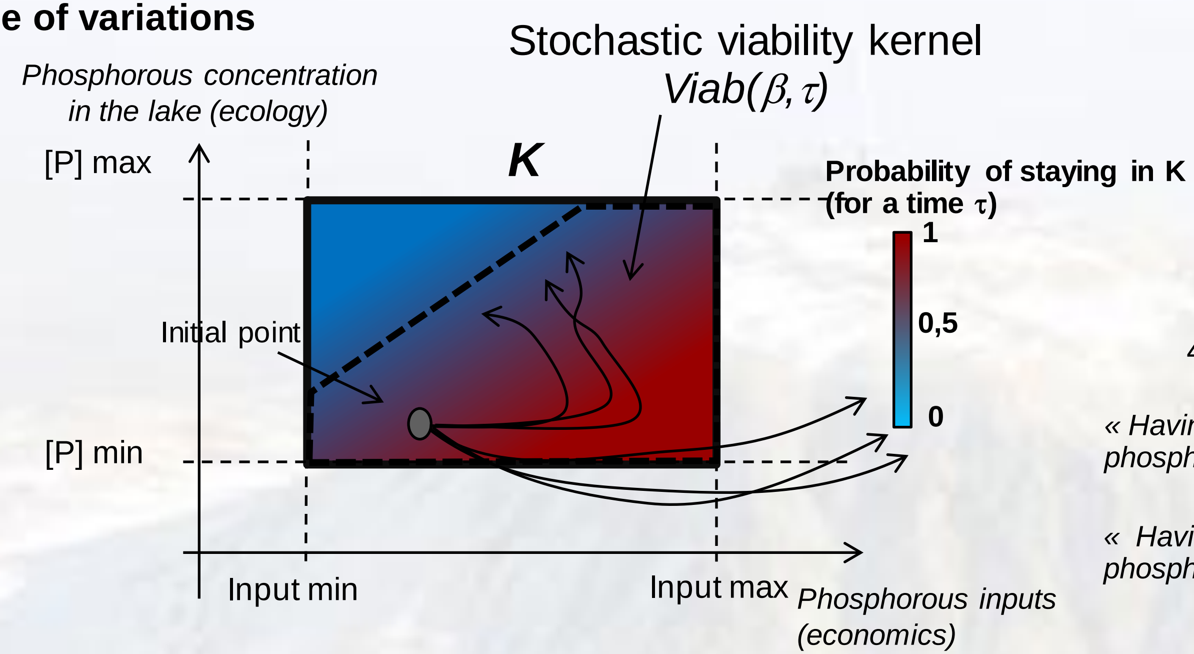


Source : CISALB - Observatoire écologique 2012
du bassin versant du lac du Bourget

We don’t have anymore a deterministic response…
… but a probabilistic response!

Stochastic viability kernel

Case of variations



4 properties of interest:

« Having a minimum and a maximum values of phosphorous concentration » (ecological issues)

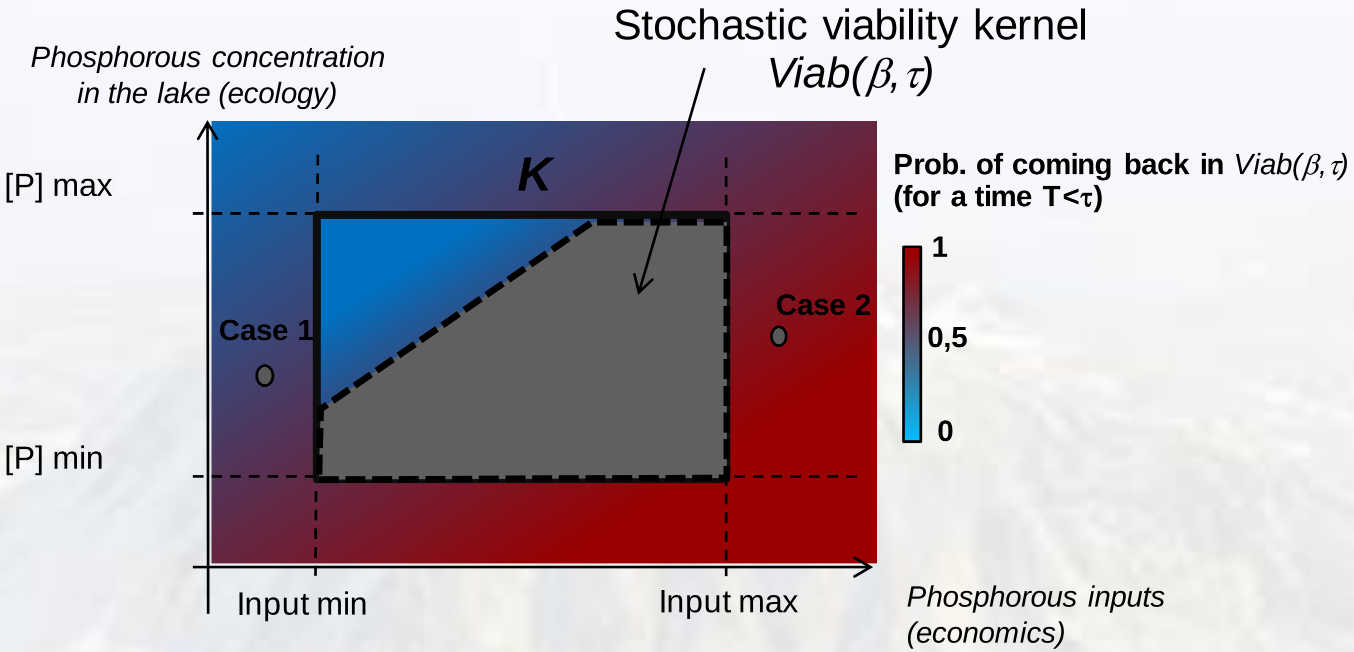
« Having a minimum and a maximum values of phosphorous inputs » (economic issues)

Viab(β,τ) is the set of initial points (in ***K***) such as there is at least a policy that enables to stay in ***K*** for a time τ, with a probability higher than β

$$\mathrm{Viab}(\beta,\tau) = \{\ x_0 \in K \mid \exists f \in F(\tau),\ \mathbb{P}\left(\forall t \in [0,\tau], x_t = g_f(t,x_0) \in K\right) \geq \beta\ \}$$

Resilience 2.0, stochastic case: probability of resilience

Case of a « surprise »



The probability of resilience corresponds to the probability to come back in *Viab(β, τ)* for given time horizonT<τ (PhD of Charles Rougé).

Optimization of policy actions

Outline

I. Concept of resilience

II. Viability theory

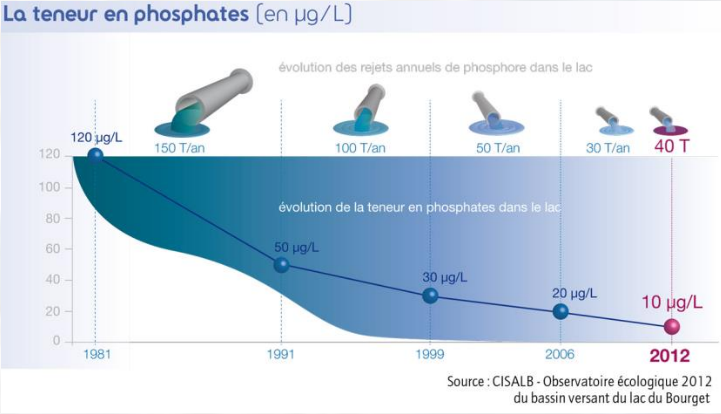
III. Case of lake eutrophication

IV. Conclusions

III. Case of lake eutrophication

Lake eutrophication

Lake Aydat



Source : CISALB - Observatoire écologique 2012 du bassin versant du lac du Bourget

Modeling the dynamics of the phosphorous concentration P (based on (Carpenter *et al.*, 1999))

$$\begin{cases} \dfrac{dP}{dt} = \underbrace{-P}_{\textbf{output}} \mathbin{\textbf{+}} \underbrace{+\,L^* + \varepsilon}_{\textbf{input}} \mathbin{\textbf{+}} \underbrace{+\,r\dfrac{P^8}{P^8+1}}_{\textbf{recycling}} \\[2em] \dfrac{dL^*}{dt} = u \qquad u \in [U^{min}, U^{max}] \end{cases}$$

We can act on the inputs

Defining viable and resilient (2.0) policies

Properties of interest?

Lake Aydat

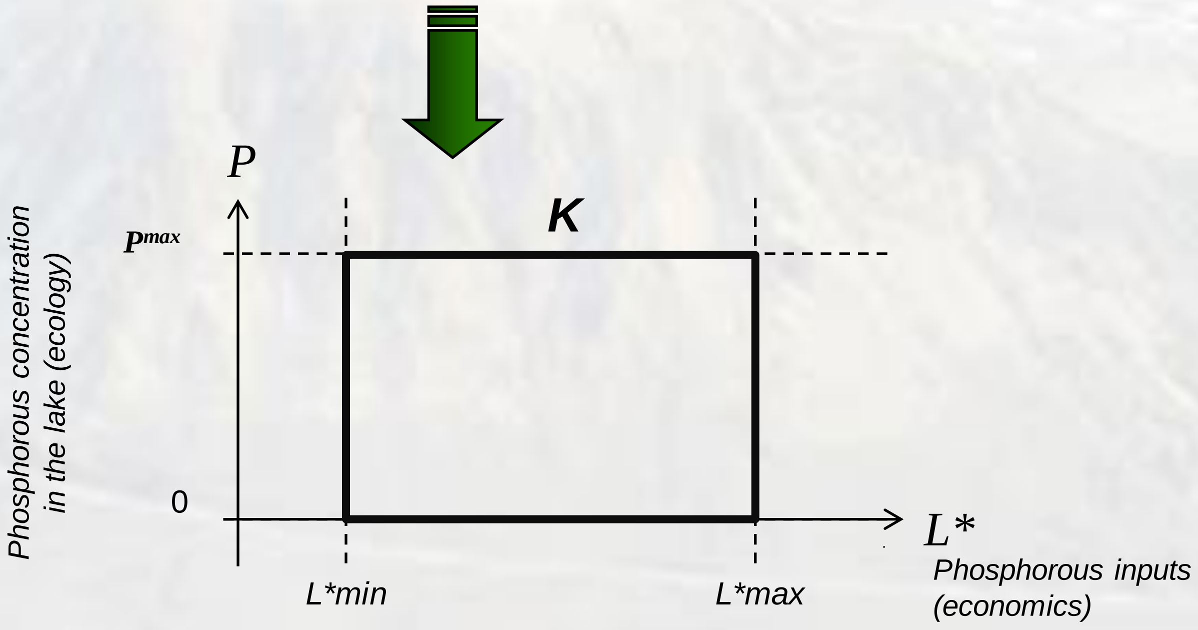
$$\begin{cases} \frac{dP}{dt} = \underbrace{-P}_{\text{output}} + \underbrace{L^*}_{\text{input}} + \underbrace{\varepsilon + r\frac{P^8}{P^8+1}}_{\text{sediments}} \\ \frac{dL^*}{dt} = u \qquad u \in [U^{min}, U^{max}] \end{cases}$$

We can act on the inputs

3 properties of interest:

« Having a maximum values of phosphorous concentration » (ecological issues)

« Having a minimum and a maximum values of phosphorous inputs » (economic issues)



Stochastic viability kernel

Lake Aydat

$$\begin{cases} \dfrac{dP}{dt} = \underbrace{-P}_{\text{output}} + \underbrace{L^* + \varepsilon}_{\text{input}} + \underbrace{r\dfrac{P^8}{P^8+1}}_{\text{sediments}} \\ \dfrac{dL^*}{dt} = u \qquad u \in [U^{min}, U^{max}] \end{cases}$$

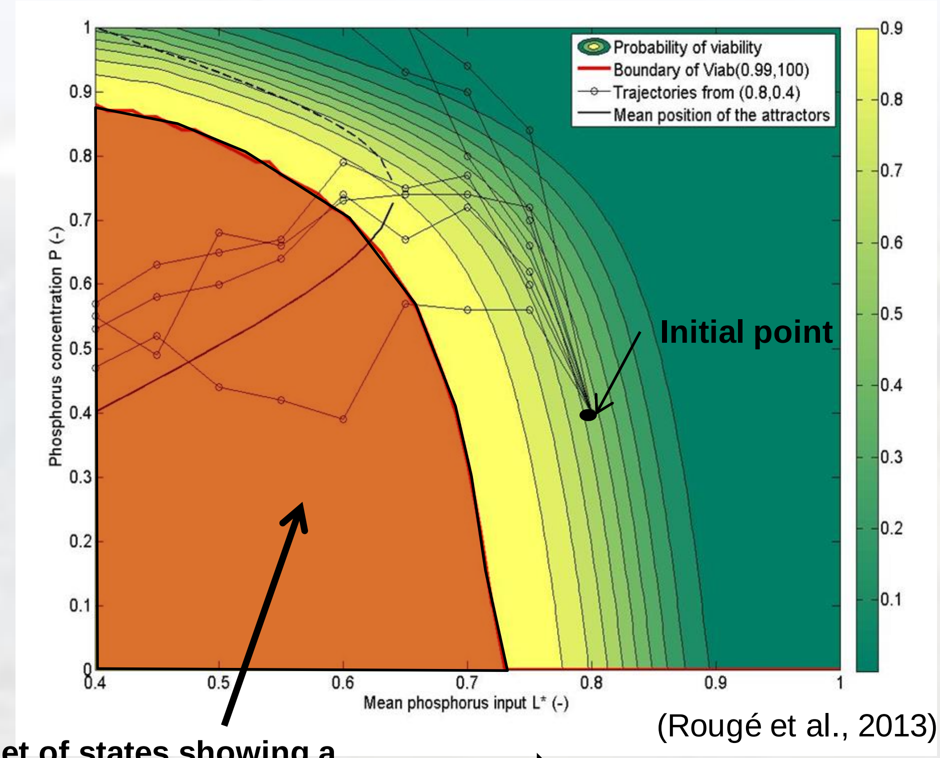
We can act on the inputs

3 properties of interest:

« Having a maximum values of phosphorous concentration » (ecological issues)

« Having a minimum and a maximum values of phosphorous inputs » (economic issues)

Time horizon= 100, ε=*N(0,0.2)*



Initial point

(Rougé et al., 2013)

Set of states showing a probability>0.95 of keeping the properties despite of variations → **Robust states???**

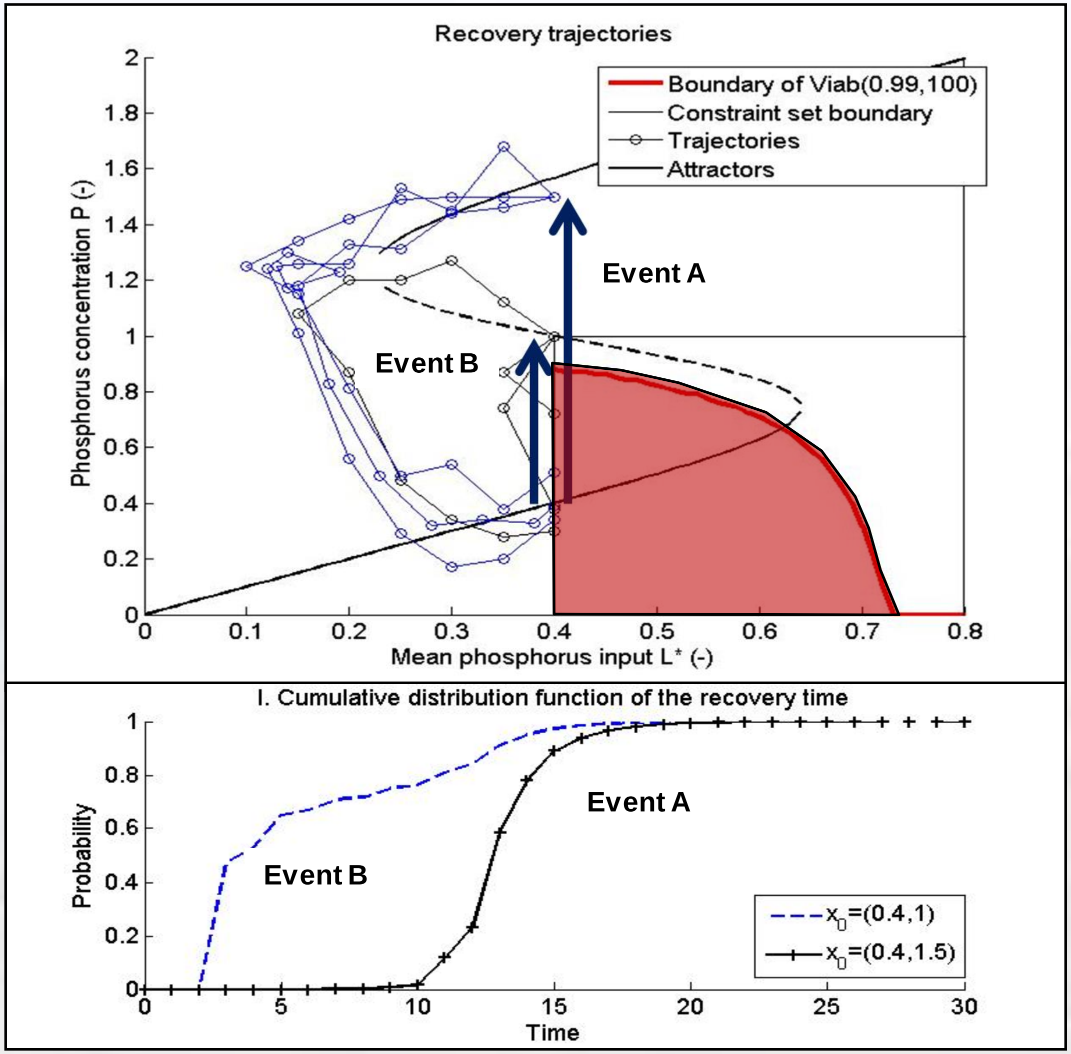
III. Case of lake eutrophication

Resilience

Lake Aydat

$$\begin{cases} \dfrac{dP}{dt} = \underbrace{-P}_{\text{output}} + \underbrace{L^* + \varepsilon}_{\text{input}} + \underbrace{r\dfrac{P^8}{P^8+1}}_{\text{sediments}} \\ \\ \dfrac{dL^*}{dt} = u \qquad u \in [U^{min}, U^{max}] \end{cases}$$

We can act on the inputs



(Rougé et al., 2013)

Outline

I. Concept of resilience

II. Viability theory

III. Case of lake eutrophication

IV. Conclusions

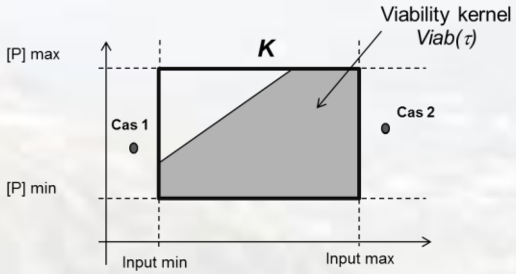
IV Conclusion

Take-home messages

Resilience 1.0 *vs* resilience 2.0

- Recover properties of interest (1.0) and keeping it (2.0)

Using the viability kernel (through the viability theory)



Case of variations (probabilistic approach)

- Probability of keeping a given property (for a given time horizon);
- Probability of recovering in this robust set (for a given time horizon).

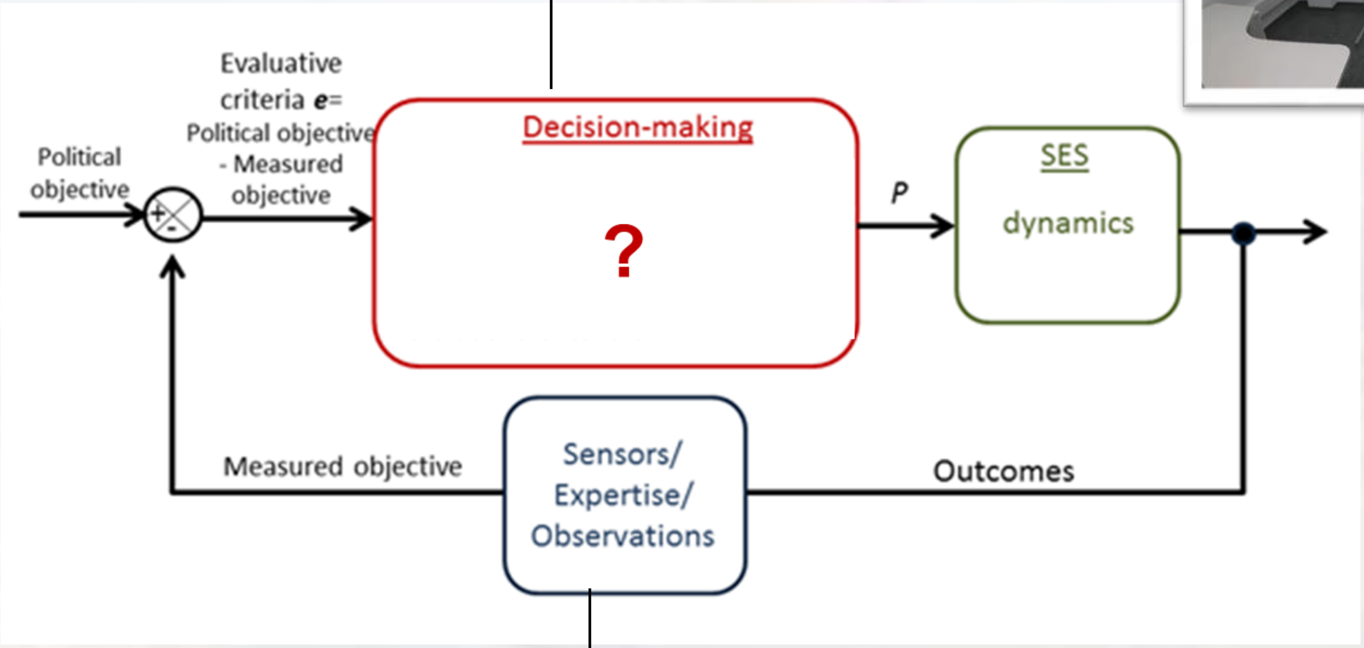
Formalization of concepts

- Vulnerability;
- Reliability;
- Flexibility;
- Capacity of adaptation.

Perspectives

Using methods from control engineering for managing social-ecological systems

Learning, adaptive management, knowledge



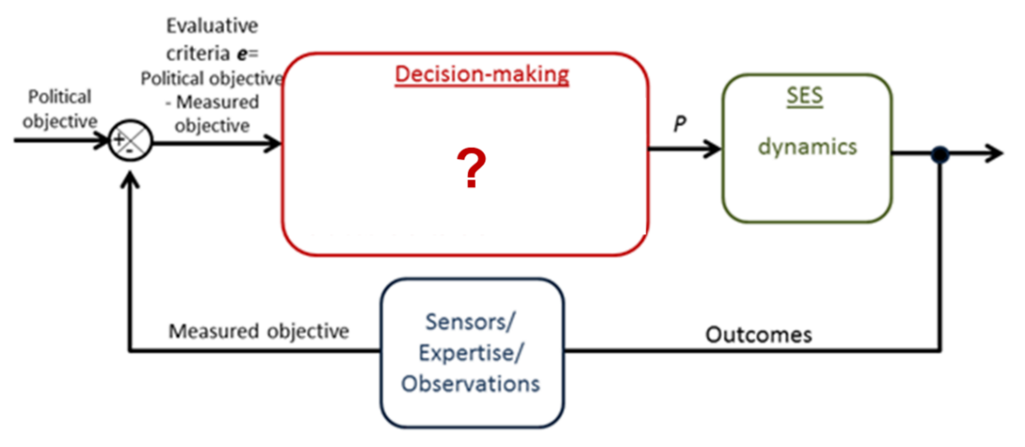
Qualitative data (surveys, narratives…)

IV Conclusion

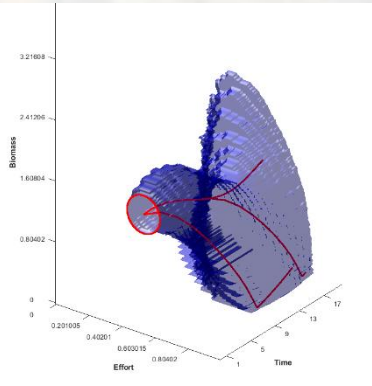
Resilience 1.0 or 2.0?

Perspectives

Management of social-ecological systems



Improving algorithms
D

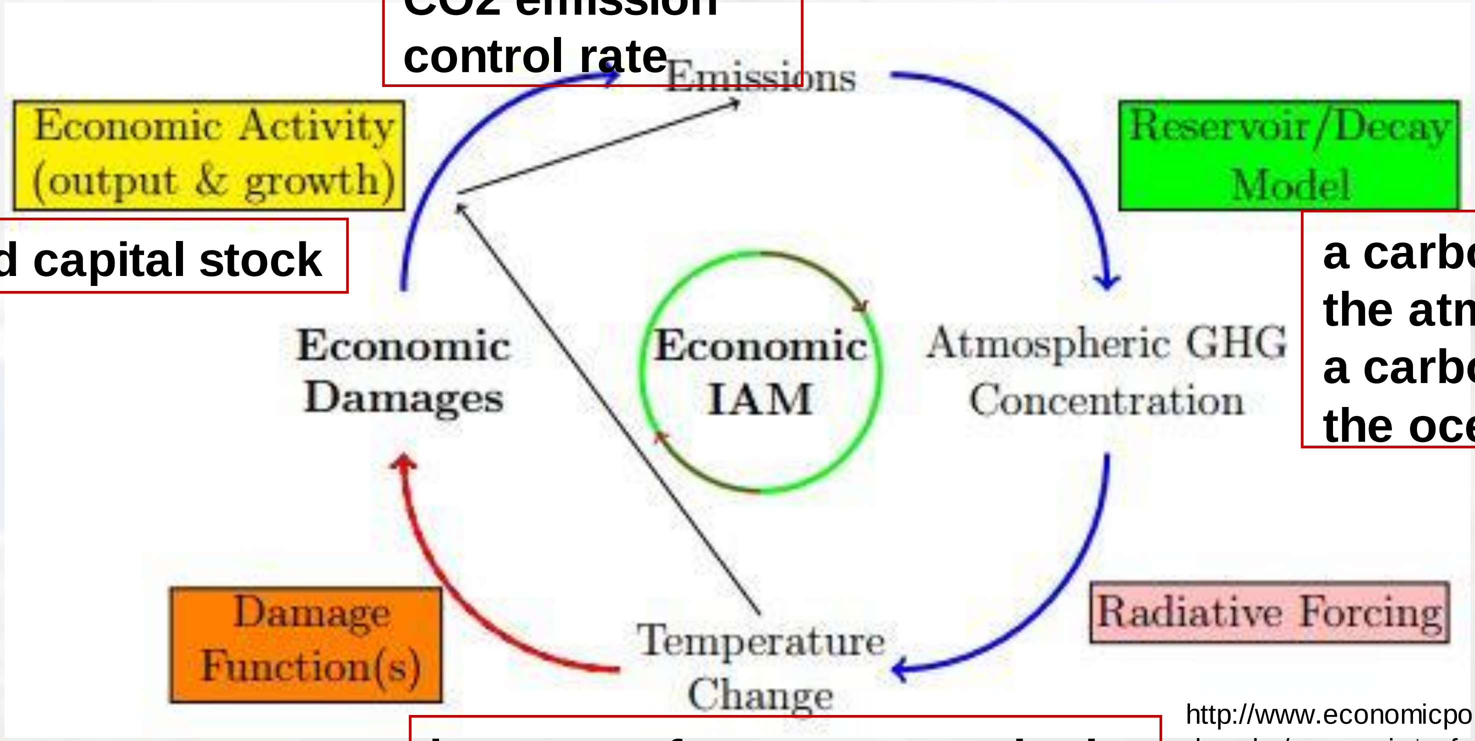


Illustration

DICE model

(Nordhaus)

Modèle d'intéraction climate-économie



CO2 emission control rate

World capital stock

a carbon stock in the atmosphere, a carbon stock in the ocean

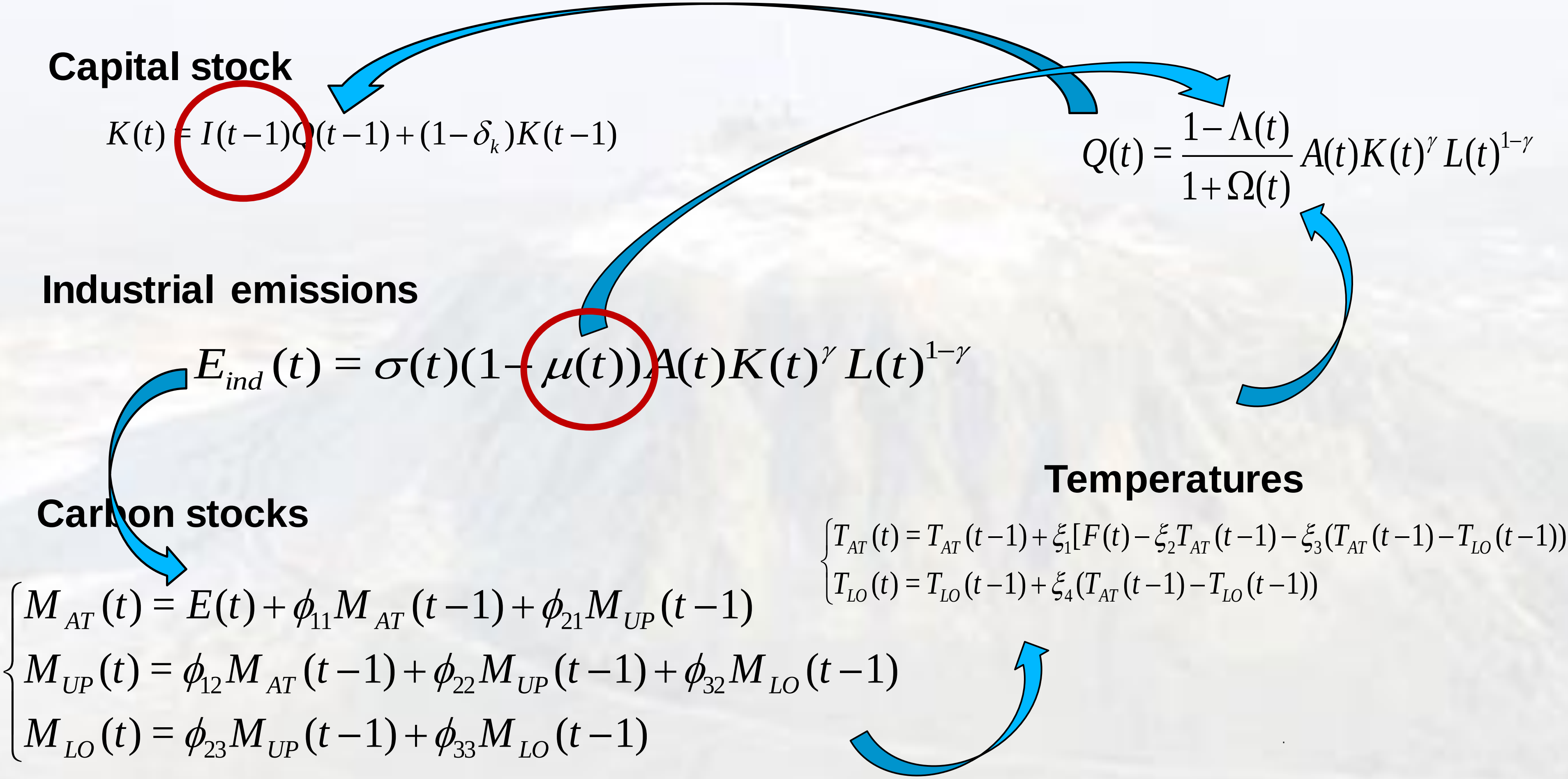
increase of temperatures in the atmosphere and in the ocean

http://www.economicpolicyresearch.org/index.php/economics-of-climate-change-news/1447-modeling-economic-damages-from-climate-change

Illustration

DICE model

(Nordhaus)



Capital stock

$$K(t) = \boxed{I(t-1)}Q(t-1) + (1-\delta_k)K(t-1)$$

$$Q(t) = \frac{1-\Lambda(t)}{1+\Omega(t)} A(t)K(t)^{\gamma} L(t)^{1-\gamma}$$

Industrial emissions

$$E_{ind}(t) = \sigma(t)(1-\boxed{\mu(t)})A(t)K(t)^{\gamma} L(t)^{1-\gamma}$$

Temperatures

$$\begin{cases} T_{AT}(t) = T_{AT}(t-1) + \xi_1[F(t) - \xi_2 T_{AT}(t-1) - \xi_3(T_{AT}(t-1) - T_{LO}(t-1)) \\ T_{LO}(t) = T_{LO}(t-1) + \xi_4(T_{AT}(t-1) - T_{LO}(t-1)) \end{cases}$$

Carbon stocks

$$\begin{cases} M_{AT}(t) = E(t) + \phi_{11} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1) \\ M_{UP}(t) = \phi_{12} M_{AT}(t-1) + \phi_{22} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1) \\ M_{LO}(t) = \phi_{23} M_{UP}(t-1) + \phi_{33} M_{LO}(t-1) \end{cases}$$

Illustration

Observabilité et controllabilité de systèmes socio-écologiques et PID

Cas « changement climatique »

Dynamique:

$$K(t+\Delta t) = 1.25\frac{1-a(t)}{1+d(t)}p(t)K(t)^{0.3}l(t)^{0.7} + 0.9^5K(t)$$
$$C(t) = [1-\mu(t)]p(t)K(t)^{0.3}l(t)^{0.7} + 0.9120C(t-\Delta t) + 70$$

Objectif: 350ppm

Observation: 1/ le CO2 OU 2/ le coût économique

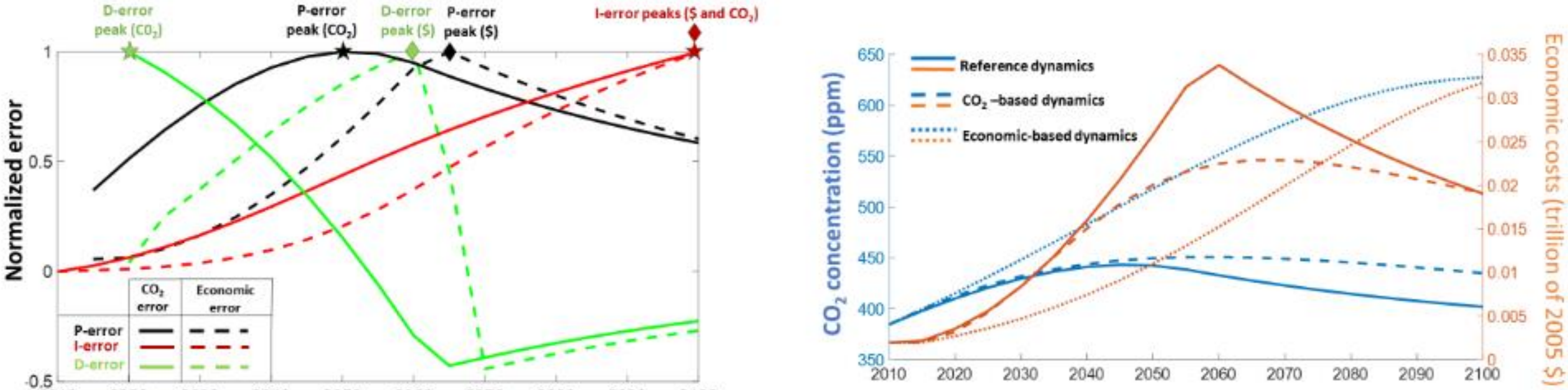
Erreur: 1/ [C02]-350 OU 2/ Coût économique

Contrôle: $\mu(t) = \mu(t-1) + \gamma_P e(t-1) + \gamma_I \sum e(t-i\Delta t)\Delta t + \gamma_d\,(e(t)-e(t-1))/\Delta t$

Illustration

Observabilité et controllabilité de systèmes socio-écologiques et PID

Cas « changement climatique »



a - Errors, reference case

b - Co2 and economic dynamics